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COMPUTER UTILIZATION IN STRUCTURAL ENGINEERING

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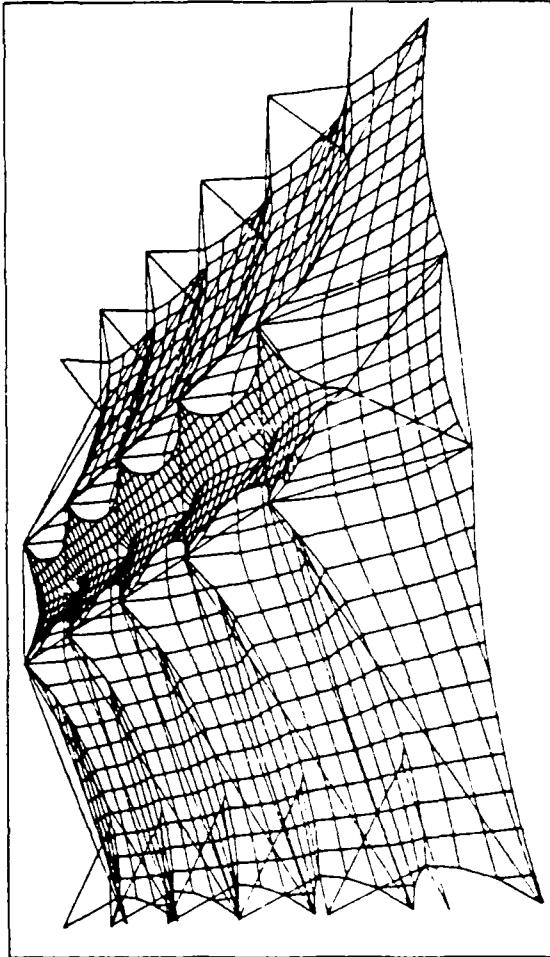
ANALYSIS OF THE NON-PLANAR RESPONSE OF A CANTILEVER, WITH THE AID OF COMPUTERIZED SYMBOLIC MANIPULATION

M.R.M. Crespo da Silva*

ABSTRACT

The nonlinear differential equations governing the static deformations of a cantilever with a tip mass are formulated and analyzed with the aid of computerized symbolic manipulation. The nonlinearities in the equations are geometric and include terms such as nonlinear contributions to the curvature expression. All the algebra in this work is relegated to the computer.

INTRODUCTION

Most of the work presented in the literature dealing with the response of structural elements such as beams is restricted to planar response. In addition, much of the work is also restricted to the cases where the deformations are very small and are analyzed by linear theory. In general, a structure can undergo flexure in any direction in space, and torsion. For infinitesimally small deformations, flexural and torsional motions are uncoupled. In this paper the flexural-flexural-torsional deformations of a cantilever with a tip mass are determined. The analysis makes use of computerized symbolic manipulation.

Computerized symbolic manipulation allows the analyst to relegate tedious algebra to a computer and, thus, to concentrate efforts in the basic steps of the formulation and analysis of problems. MACSYMA (Rand, 1984) is among the most powerful of such A.I. programs. It can perform differentiations and integrations in analytic form, Taylor series expansions, solve algebraic and differential equations, etc. Here the nonlinear differential equations governing the flexural-flexural-torsional static response of a cantilever with a tip mass are formulated and analyzed with the aid of MACSYMA. The equations, and their boundary conditions, are formulated via Hamilton's principle. To consider the general case where the beam is able to bend in any direction in space and twist, a set of angles to describe the orientation of an arbitrary cross section of the beam is used. The equations are written in terms

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of the deflections relative to a set of inertial reference axes and of a torsional variable. Subsequently they are converted into a set of integro-partial differential equations in the bending deflections by eliminating the remaining elastic variables. For simplicity the material is assumed to be Hookean, and the beam's properties to be constants. The nonlinearities in the equations are geometric and include terms such as nonlinear contribution to the curvature expression. By expanding in Taylor series the nonlinearities in the equations, a set of equations with polynomial nonlinearities is generated. A perturbation analysis of the response is then performed using such equations to obtain an analytical approximation for the beam's deflections. This is also done with the aid of MACSYMA. All the tedious algebra involved in the formulation is relegated to the computer which displays the requested equations on the screen.

PROBLEM FORMULATION

Consider an initially straight and untwisted, clamped-free beam of length ℓ, mass m per unit length, and with a lumped mass M, of weight Mg, located at its tip s=1. Here, s denotes arc-length along the beam, non-dimensionalized by ℓ. Figure 1a shows a beam element of length s in its deformed state. The position vector of the centroid C of the beam's cross section S, normal to the tangent line at s=s, is written in terms of the elastic deformation components of C, u(s), v(s) and w(s), as $\bar{r}_C = [s+u(s)]\hat{x} + v(s)\hat{y}+w(s)\hat{z}$. The unit vectors $(\hat{x},\hat{y},\hat{z})$ are inertial. The mass applied at the beam's tip at s=1 is $Mg[\hat{y}\cos\alpha+\hat{z}\sin\alpha]$, where α is the angle from the vertical axis to the direction defined as $\hat{y}=\hat{\eta}(s=0)$. The unit vectors $(\hat{\xi},\hat{\eta},\hat{\zeta})$ are aligned with the principal axes of the beam's cross at s=s. The orientation of the axes (ξ,η,ζ), relative to the inertial axes (x,y,z) shown in Figure 1a, can be described by the three successive rotations (ψ,θ,ϕ) shown in Figure 1b. Here, $( )'=d( )/ds$.
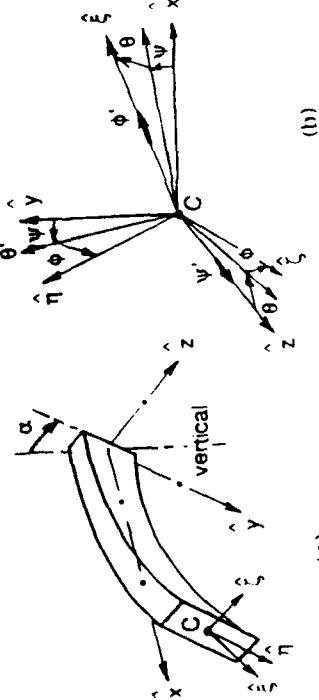
Figure 1. Beam element, and the orientation angles (ψ,θ,ϕ)

By neglecting the effect of shear, it is seen that the angles ψ and θ are related to the derivatives of the elastic deflections as, $\tan\psi=v'/(1+u')$ and $\tan\theta=-w'/[(1+u')^2+v'^2]^{1/2}$. By approximating the beam as inextensional, it follows that $(1+u')^2+v'^2+w'^2=1$. The differential equations governing the elastic deformations of the beam can be obtained either by a vectorial formulation or by a variational approach. Either approach will give equivalent equations, of course, and the use of one approach over the other is basically a matter of individual preference. We prefer to use the variational formulation, which also gives a boundary condition equation. For simplicity we will assume that the beam's material is Hookean, and that all the beam properties are constants. Letting D_η and D_ζ denote the beam's principal bending stiffnesses, and D_ξ denote its torsional stiffness, the specific Lagrangean for the system, normalized by D_η/ℓ, is simply $L=-(\beta_\xi\rho_\xi^2+\rho_\eta^2+\beta_\zeta\rho_\zeta^2)/2$, where $(\rho_\xi,\rho_\eta,\rho_\zeta)$ are the components of the curvature vector at C (including the torsion), along $(\hat{\xi},\hat{\eta},\hat{\zeta})$, and $\beta_\xi=D_\zeta/D_\eta$ and $\beta_\eta=D_\zeta/D_\eta$. For simplicity it will be assumed that $M>>m\ell$ so that the static deflection due to the weight of the beam is neglected. The differential equations of equilibrium can be formulated from Hamilton's principle, with the constraint condition adjoined to the Lagrangean with a Lagrange multiplier λ, as (Crespo da Silva and Glynn, 1978a)

$$\delta I = \delta \int_0^1 \left\{ \int \left[L(\psi,\theta,\phi,\psi',\theta',\phi') + \frac{\lambda}{2}\left[1-(1+u')^2 - v'^2 - w'^2\right]\right] ds \right.$$
$$\left. + \frac{Mgl^2}{D_\eta}\left[(\cos\alpha)\delta v + (\sin\alpha)\delta w\right]_{s=1} \right\} = 0 \tag{1}$$

The vector $\bar{\rho}$ is obtained directly from Figure 1b as,

$$\bar{\rho} = (\phi'-\psi'\sin\theta)\hat{\xi} + (\psi'\cos\theta\sin\phi+\theta'\cos\phi)\hat{\eta}$$
$$+ (\psi'\cos\theta\cos\phi - \theta'\sin\phi)\hat{\zeta} = \rho_\xi\hat{\xi} + \rho_\eta\hat{\eta} + \rho_\zeta\hat{\zeta} \tag{2}$$

By performing the variations indicated in Eqn.(1), the following differential equations are obtained,

$$G'_u - \left[A_v\frac{\partial v}{\partial u'} + A_\theta\frac{\partial v}{\partial u'} + \Lambda(1+u')\right]' = 0 \tag{3a}$$

$$G'_k - \left[A_v\frac{\partial v}{\partial u'} + A_\theta\frac{\partial v}{\partial u'} + \Lambda k'\right]' = 0 \quad ; \quad k=v,w \tag{3b}$$

$$\Lambda_b = 0 \tag{3c}$$

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where $A_k = (\partial L/\partial k')' - \partial L/\partial k$ for $k = \psi,\theta,\phi$. The following boundary condition equation is also obtained from Eqn.(1).

$$\{B_\eta(\phi'-\psi'\sin\theta)\delta\phi+G_u\delta u+(G_v-\iota P\cos\alpha)\delta v+(G_w-\iota P\sin\alpha)\delta w$$
$$-\left[H_v-\frac{v'H_u}{1+u'}\right]\delta v'-\left[H_w-\frac{u'H_u}{1+u'}\right]\delta u'\}\Big|_{s=1} = 0 \quad;\quad \iota = 0,1 \qquad (4)$$

where $P=Mg\ell^2/D_\eta$, and $H_k = (\partial L/\partial u')\partial u'/\partial k' + (\partial L/\partial \theta')\partial \theta'/\partial k'$ for $k=u,v,w$.

RESPONSE ANALYSIS

The differential equations (3a-c) are nonlinear and coupled. To find an approximate solution for the elastic deformations $v(s)$ and $w(s)$, and for the orientation angle $\phi(s)$, we will expand these equations in Taylor series about $v(s)=w(s)=\phi(s)=0$ and then solve the resulting equations by a perturbation technique. For this we let $k(s)=\epsilon k^*(s)$, for $k=v,w$ and ϕ, where ϵ is a small arbitrary parameter which is used for bookkeeping purposes only. Here we will obtain an approximate solution to the elastic deformations to order ϵ^3. Since this involves a large amount of algebra, MACSYMA is used to develop all the variational equations and to perform the perturbation analysis. MACSYMA is a very large symbolic manipulation program for applied mathematics. It also includes a number of routines for numerical computation and for generating plots. MACSYMA runs on SUN, VAX and Symbolics computers, and on a few other machines. There are a number of other smaller programs that also run on these and other machines and, in some cases, on mainframes and personal computers. Among these are REDUCE (Hearn,1973), MAPLE (Char, Geddes, Gonnet, Monagan and Watt, 1988) and muMATH (Wooff and Hodgkinson, 1987). Many of MACSYMA commands are self-explanatory, such as (C1)TAYLOR(sin(epx*x)*sqrt(1-epx**2*x**2), eps, 0, 10), for the Taylor series of the first argument, in EPS, about EPS=0, to EPS to the 10th power; (C2)DIFF(sin(epx*x)*sqrt(1-epx**2*x**2),x,5), for the fifth derivative, with respect to x, of the given expression;(C3)INTEGRATE(sin(epx*x)*sqrt(1-eps**2*x**2), x, 0, 1), for the integral of the given expression, in x, from x=0 to x=1;(C4)PART([A1,A2,A3],3),for the third part of the list [A1,A2,A3]; (C5)LHS(y= x**2-3) for the left-hand side (or part 1), of the given equation;(C6)SOLVE([EQN1,EQN2],[x,y]) to obtain the solution of the system of equations EQN1=0 and EQN2=0 for the variables x and y; etc.For each Ci command, MACSYMA generates a Di output with the result of the computation. The computer generated equations are displayed as if they were written by a mathematician.

To analyze the problem considered here, we begin by telling MACSYMA that each angle in the list [ψ,θ,φ] de-

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pends on the independent variable s, and then define the components of the vector ρ as shown below. MACSYMA's response to the command C3 is shown in the line labelled D3.

(C1) list_of_angles : [psi,theta,phi]$
(C2) depends(list_of_angles, s)$
(C3) [rhoxi:diff(phi,s)-diff(psi,s)*sin(theta),
rhoeta:diff(psi,s)*cos(theta)*sin(phi) + diff(theta,s)*cos(phi),
rhozeta:diff(psi,s)*cos(theta)*cos(phi) - diff(theta,s)*sin(phi)];

$$(D3)\quad \left[\frac{d\,phi}{ds}-\frac{d\,psi}{ds}\sin(theta),\right.$$
$$\cos(phi)\frac{d\,theta}{ds}+\sin(phi)\frac{d\,psi}{ds}\cos(theta),$$
$$\left.\cos(phi)\frac{d\,psi}{ds}\cos(theta)-\sin(phi)\frac{d\,theta}{ds}\right]$$

Next, the normalized Lagrangean L is defined and the expressions for A_i, defined in the previous section for $i=\psi,\theta$ and ϕ, are developed as

(C4) L:-(betag*rhoxi**2+rhoeta**2+betay*rhozeta**2)/2$
(C5) for i in list_of_angles do (Ai:concat(A,i),
Ai::diff(diff(L,diff(i,s)),s)-diff(L,i))$

As shown in the previous section, the nonlinear differential equations of equilibrium are obtained as $A_\phi=0$ and $G_i'=0$, for $i=u,v$ and w. To let MACSYMA develop the expressions for G_i, the variables u,v and w are declared to be dependent on s, and the constraint relationships for the angles ψ and θ are introduced as shown in C8.

(C6) list_of_vars:[u,v,w]$ (C7) depends(list_of_vars, s)$
(C8) lpsi:atan(diff(v,s)/(1+diff(u,s))),
theta:atan(diff(w,s)/sqrt((1+diff(u,s))**2 + diff(v,s)**2))]$

To obtain a set of differential equations with polynomial nonlinearities, a small arbitrary bookkeeping parameter ϵ is now introduced so that $v=O(\epsilon)$, $w=O(\epsilon)$ and $\phi=O(\epsilon)$.To order ϵ^3, the inextensionality constraint yields a simple expression for u', as $u'=-(v'^2+w'^2)/2$. This information is given to MACSYMA in the following lists, which will be used when generating the expanded forms of G_u, G_v, G_w and A_ϕ.

(C9) list:1:[u=eps**2*u(s),v=eps*v(s),w=eps*w(s),phi=eps*phi(s)]$
(C10) list:2:[diff(u,s)=-diff(v,s)**2/2-diff(w,s)**2/2]$

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The expanded forms of G_u, G_v and G_w can be generated with the following sequence of commands grouped in a loop.

```
(C11) for i in list_of_vars do ( print("evaluating",G:concat(G,i)),
auxl:if i=u then 1+diff(u,s) else diff(i,s), aux2:ev( Apsi*diff(
psi, diff(i,s)) + Atheta*diff( theta,diff(i,s)) ) + lambda*aux1,aux2:
ev(aux2,list1,list2,diff), print("doing the Taylor series expansion"),
Gi::taylor(aux2,eps,0,3) )$
```

Since the Lagrange multiplier λ is only needed to $O(\epsilon^2)$, as disclosed by Eqn. (3b), the differential equation $G_u'=0$ is then solved for λ, with the boundary condition $G_u(s=1)=0$. This is simply done with the command C12 shown below. The expression obtained for λ is shown in D12 (the symbols ϵ, β_y and β_7, instead of EPS, betay and betag, are used in the versions of the displays shown here).

(C12) lambda:rhs(part(taylor(solve(gu,lambda),eps,0,2), 1)):

$$(D12) \quad \left(-\beta_y \frac{d^3v(s)}{ds^3} \frac{dv(s)}{ds} - \frac{d^3w(s)}{ds^3} \frac{dw(s)}{ds} \right) \epsilon^2 + \cdots$$

The expanded form of A_ϕ is generated as shown below. The $O(\epsilon^3)$ expression for A_ϕ is shown in D14.

(C13) Aphi:ev(Aphi)$
(C14) Aphi:taylor(ev(Aphi, list1, list2, diff), eps, 0, 3);

$$(D14) \quad -\beta_7 \frac{d^2phi(s)}{ds^2} \epsilon + \left((\beta_y - \beta_7 - 1) \frac{d^2v(s)}{ds^2} \frac{d^3v(s)}{ds^3} - \beta_7 \frac{d^3v(s)}{ds^3} \frac{dw(s)}{ds} \right) \epsilon^2$$
$$+ (\beta_y - 1)phi(s) \left(\left(\frac{d^2w(s)}{ds^2} \right)^2 - \left(\frac{d^2v(s)}{ds^2} \right)^2 \right) \epsilon^3 + \cdots$$

The above expression for A_ϕ discloses that $\phi=O(\epsilon^2)$. The expression for A_ϕ is then truncated to $O(\epsilon^2)$ --- command C15 -- and the differential equation $A_\phi=0$ is solved for ϕ as shown in C17 and C18. The resulting expression for ϕ is given in D18.

(C15) Aphi:subst((eps**3=0), Aphi)$
(C15) declare(integrate, additive)$
(C17)phi_double_prime:rhs(part(solve(Aphi,diff(phi(s),s,2)),1))$
(C18)phi_of_s:integrate(integrate(phi_double_prime,s,1,s), s, 0,s);

$$(D18) \quad \left(\frac{\beta_y - \beta_7 - 1}{\beta_7} \int_0^s \int_1^s \frac{d^2v(s)}{ds^2} \frac{d^2w(s)}{ds^2} ds ds - \int_0^s \int_1^s \frac{d^3v(s)}{ds^3} \frac{dw(s)}{ds} ds ds \right) \epsilon$$

The variable ϵ is now eliminated in the expressions obtained above for G_v and G_w. This is done by re-evaluating those expressions with the MACSYMA command ev to replace ϕ by the expression phi_of_s given by D18, and forcing MACSYMA to carry out all the intermediate differentiations. As a result, two nonlinear integro-partial differential equations are now obtained in the variables $v(s)$ and $w(s)$. The integro-differential expressions for G_v and G_w are obtained as shown below. The simplified form of the expression obtained for G_v is displayed in D21.

(C19) Gv: expand(ev(gv, [phi(s) = phi_of_s], diff)) $
(C20) Gw: expand(ev(gw, [phi(s) = phi_of_s], diff)) $
(C21) ratsimp(Gv);

$$(D21) \quad -\left((\beta_y^2 - (\beta_7 + 2)\beta_y + \beta_7 + 1) \epsilon^3 \frac{d^3u(s)}{ds^3} \epsilon^3 \int_0^s \int_1^s \frac{d^2v(s)d^2w(s)}{ds^2} ds ds \right.$$
$$+ \beta_7(1-\beta_y)\epsilon^3 \frac{d^3w(s)}{ds^3} \int_0^s \int_1^s \frac{d^3v(s)}{ds^3} \frac{dw(s)}{ds} ds ds$$
$$+ (\beta_y^2 - 2\beta_y - \beta_7^2 + 1)\epsilon^3 \frac{d^2w(s)}{ds^2} \int_1^s \frac{d^2v(s)}{ds^2} \frac{d^2u(s)}{ds^2} ds$$
$$+ \beta_7\beta_y \epsilon^3 \frac{dv(s)}{ds} \frac{dw(s)}{ds} \frac{d^3w(s)}{ds^3} + \beta_7\beta_y \epsilon^3 \frac{dv(s)}{ds} \frac{d^3v(s)}{ds^3} \left(\frac{d^2u(s)}{ds^2} \right)^2$$
$$+ (\beta_7\beta_y + \beta_y^2 - \beta_7)\epsilon^3 \frac{d^2w(s)}{ds^2} \left(\frac{dw(s)}{ds} \frac{d^2v(s)}{ds^2} \right) - \int_1^s \frac{d^3v(s)}{ds^3} \frac{dw(s)}{ds} ds$$
$$+ \beta_7\beta_y \left(\epsilon^3 \left(\frac{dv(s)}{ds} \right)^2 + \epsilon \right) \frac{d^3v(s)}{ds^3} + \beta_7\beta_y \epsilon^3 \frac{d^3v(s)}{ds^3} \left(\frac{d^2v(s)}{ds^2} \right)^2 \right) /\beta_7$$

A few integrations by parts in the new expressions for G_v and G_w disclose that they are the same as those obtained in (Crespo da Silva and Glynn, 1978b). Since the boundary conditions for the differential equations $G_v'=0$ and $G_w'=0$ are $G_v(s=1)=P\cos\alpha$ and $G_w(s=1)=P\sin\alpha$. For $v=O(\epsilon)$ and $w=O(\epsilon)$, it follows that the $O(\epsilon)$ part of each of these equations is

(C22) [coeff(Gv=eps*P*cos(alpha), eps),
      coeff(Gw=eps*P*sin(alpha), eps)];

$$(D22) \quad \left[-\beta_y \frac{d^3 v(s)}{ds^3} = P\cos\alpha, \quad -\frac{d^3 w(s)}{ds^3} = P\sin\alpha \right]$$

The solution to these O(ϵ) differential equations is simply $v(s) \equiv v_1 = -pv(s^3 - 3s^2)/6$ and $w(s) \equiv w_1 = -pw(s^3 - 3s^2)/6$ where $pv = P\cos\alpha/\beta_y$ and $pw = P\sin\alpha$, i.e.

(C23) [v1:-pv*(s**3-3*s**2)/6, w1:-pw*(s**3-3*s**2)/6]$

To obtain an approximate solution for v(s) and w(s) we then let $v(s) = v_1 + \epsilon^2 v_3$ and $w(s) = w_1 + \epsilon^2 w_3$, substitute these expressions in the integro-differential equations of equilibrium and truncate the result to O(ϵ^3) (see C27 and C28), and then do a series of integrations to solve for v_3 and w_3 (C29 through C32). This is done by MACSYMA as shown below. The solution obtained for v3 is shown in D33.

(C24) list4:[v(s)=v1+eps**2*v3, w(s)=w1+eps**2*w3]$
(C25) [depends([v3,w3],s), assume(s>0, s<1)]$
(C26) /* The following declaration and assignment are needed to force MACSYMA to truncate the desired expressions to O(ϵ^3) */ [ratweight(eps,1), ratwtlvl:3]$
(C27) [eqn1:ev (Gv-eps*betay*pv, list4, diff, expand, ratexpand), eqn2:ev (Gw-eps*pw, list4, diff, expand, ratexpand)]$
(C28) /* force MACSYMA to do all the integrations in EQN1 and EQN2*/ [eqn1:ev(eqn1,integrate) , eqn2:ev(eqn2,integrate)]$
(C29) [v3_triple_prime:rhs(part(solve(eqn1,diff(v3,s,3)), 1)), w3_triple_prime:rhs(part(solve(eqn2, diff(w3,s,3)), 1))]$
(C30) [v3_double_prime:integrate(v3_triple_prime,s, 1,s), w3_double_prime:integrate(w3_triple_prime,s, 1,s)]$
(C31) [v3_prime:integrate(v3_double_prime,s,0,s), w3_prime:integrate(w3_double_prime,s, 0,s)]$
(C32) [v3:integrate(v3_prime,s,0,s),w3:integrate(w3_prime,s, 0,s)]$
(C33) v3:

$$(D33) \quad (((10\beta_y^2 + (23\beta_\gamma - 20)\beta_y - 5\beta_\gamma + 10)pvp_w^2 + 18\beta_\gamma\beta_yP_v^3)s^7$$
$$+ ((-70\beta_y^2 + (140 - 161\beta_\gamma)\beta_y + 35\beta_\gamma - 70)pvp_w^2 - 126\beta_\gamma\beta_yP_v^3)s^6$$
$$+ ((210\beta_y^2 + 420(\beta_\gamma - 1) - 126\beta_\gamma + 210)pvp_w^2 + 294\beta_\gamma\beta_yP_v^3)s^5$$
$$+ ((-350\beta_y^2 + (700 - 490\beta_\gamma)\beta_y + 280\beta_\gamma - 350)pvp_w^2 - 210\beta_\gamma\beta_yP_v^3)s^4$$
$$+ (280\beta_y^2 + (280\beta_\gamma - 560)\beta_y - 280\beta_\gamma + 280)pvp_w^3)s^3$$
$$+ (-168\beta_\gamma\beta_ypvp_w^2 - 168\beta_\gamma\beta_yP_v^3)s^2)/(5040\beta_\gamma\beta_y)$$

Since the labelling v(s) and w(s) should be interchangeable (see Figure 1a), one would expect that if pv is replaced by pw, D_η is replaced by D_ζ, and D_ζ is replaced by

---

D_η in the expression for w_3 (and w_1), the expression for w_3 (and w_1) should be obtained. This was verified to be true by re-evaluating v_3 with $pv \to pw$, $\beta_\gamma \to 1/\beta_y$ and $\beta_\gamma \to \beta_\gamma/\beta_y$. The O($\epsilon^3$) approximate solution for $\phi(s)$ is obtained from D?8 with v(s)=v1 and w(s)=w1.

Figure 2 shows MACSYMA-generated plots of v(s=1) and w(s=1) versus α for $\beta_y=15$, $\beta_\gamma=1.247$ and for the two values of P indicated. The parameter values indicated match the values in (Hinnant and Hodges, 1987) for a rectangular aluminum beam, with a heavy tip mass, used in experiments reported in (Dowell, Traybar and Hodges, 1977). The value of P=0.473 corresponds to a tip weight os 1(h, which is about 8 times higher than the weight of the beam. The analytical results shown in Figure ? are nearly indistinguishable from the experimental data reported in those references and from the results of a finite element analysis presented in (Hinnant and Hodges, 1987).
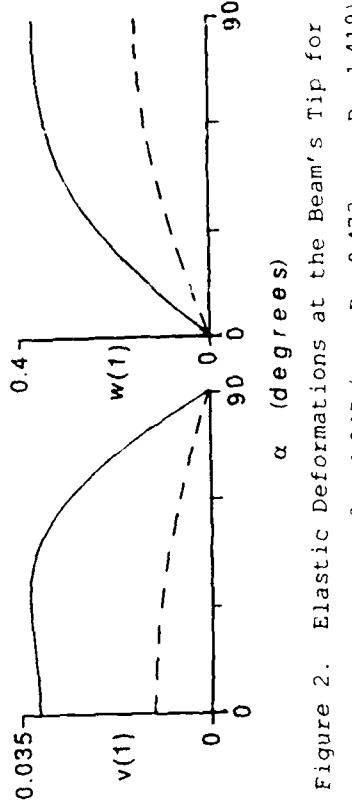

Figure 2. Elastic Deformations at the Beam's Tip for $\beta_y = 15$ and $\beta_\gamma = 1.247$ (— — — P = 0.473, ——— P = 1.419)

CONCLUSIONS

Computerized symbolic manipulation is a very useful and versatile tool for the analyst. It frees the analyst from the task of performing tedious and/or repetitive algebra, and also allows for extending analysis capabilities beyond the level that would normally be feasible by pencil and paper for a number of problems. In this paper it has been used to assist in the formulation of the nonlinear equations of equilibrium of a beam, and in the analysis of the response of the beam. Its role in rotorcraft dynamics analysis has also been discussed and demonstrated in (Crespo da Silva and Hodges, 1986). As indicated in (Hodges, Crespo da Silva and Peters, 1987), problems of this nature have been addressed by other investigators, and a number of errors appear in their analysis due, in part, to a priori unwarranted approximations and shortcuts used in order to prematurely simplify the equations.

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